Recognizing Relationships

Objectives:

- I can inspect data for two variables and identify the type of relationship (linear, quadratic, inverse, or constant) that relates the variables.
- I can distinguish between the four major types of relationships in terms of representations such as data tables, graphs, equations, and verbal statements.

Independent vs. Dependent Variables

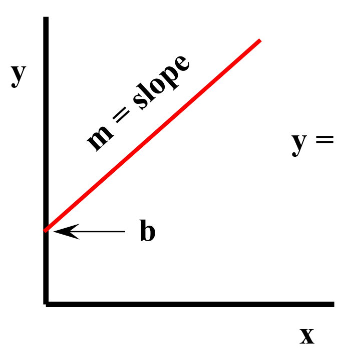
Experiments in Physics often focus on the effect that one quantity has upon another quantity. For instance, an experiment might focus on the question

How does the mass of an object effect its acceleration?

The two quantites being being studied are **mass** and **acceleration**. In experiments like these, there is an independent variable and a dependent variable. The **independent variable (IV)** is the quantity which the experimenter intentionally varies from trial to trial. The **dependent variable (DV)** is the quantity which the experimenter measures; its value always *depends* on the independent variable. In the experiment described here, the independent variable is the **mass** of the object; the experiment makes an effort to change its value from one trial to the next. The **acceleration** is the dependent variable; its value *depends* on the value of mass using in each specific trial. In general, and experimenter has the purpose of *determining the effect of changes in the independent variable upon the dependent variable.*

Linear Relationships

Studies of variables in Physics will often show the pattern of a linear relationship. As the name suggests, two variables that are linearly related will be represented by a straight line on an x-y graph. The significance of this is that any given change in the independent variable (x) will always produce the same change in the dependent variable (y). For instance, a 1-unit change in the value of x may produce a 2-unit change in the value of y; if linearly related, then this ratio of the change in y value to the change in x value is always the same ratio. Since this ratio is what we refer to as slope, then one could say that two quantities that are linearly related will result in an x-y graph that has a constant or unchanging slope. This will be demonstrated in a carefully contrived data table by the fact that the same change in x-value from one row to the another row will always result in the same change in y-value for those two respective rows. Linearly related quantities have data values that can be described by an equation of the form y = m•x + b where m is the slope of the line on the x-y plot and b is the y-intercept.

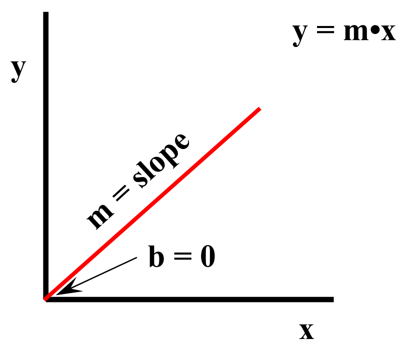


y = m•x + b

x	y
0	4
1	6
2	8
3	10
4	12
5	14

Linearly Related:
When x changes by a certain amount (1 unit), the value of y always changes by the same amount (e.g., 2 units).

A Special Type of Linear Relationship: A very common data pattern that is observed in Physics is the **directly proportional pattern**. It is a special type or sub-set of the linear relationship. Two variables that are directly proportional to one another will be represented by an x-y plot that shows a straight line (as in a linear relationship) with a y-intercept of 0. As such the typical y = m•x + b equation becomes a y = m•x equation. In such instances, a doubling of the independent variable (x) results in a doubling of the dependent variable (y). And a tripling of the independent variable causes a tripling of the dependent variable. In more general terms, the value of y will change by the same factor that the value of x is changed by. An x-y data table can be inspected to see if it follows this pattern; if it does, then the quantities x and y are both linearly related and directly proportional to one another.



x	y
0	0
1	2
2	4
3	6
4	8
5	10

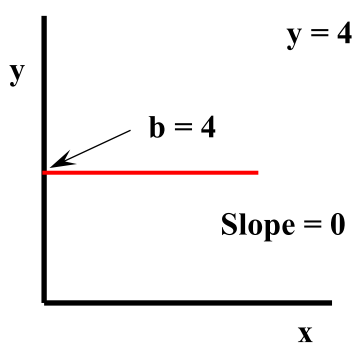
Directly Proportional

Double x … Double y

Triple x … Triple y

Non- or Constant Relationship

Sometimes a change in one variable will have no effect upon the value of the other variable. In such a situation, we would say that two variables have no cause-effect relationship between them. As the independent variable (x) is altered, there is no observable change in the value of the other variable (y). A plot of the variables x and y will demonstrate this as a horizontal line; as the value of x changes, the value of y remains constant. An x-y data table will demonstrate this non-relationship in a similar manner; the values in the x column will change but the values in the y-column will remain constant. And finally, an equation will show that y is a constant.



x	y
0	4
1	4
2	4
3	4
4	4
5	4

Non-Relationship

Changes in the value of x have no effect upon the value of y

The y value is a constant.

Quadratic Relationships

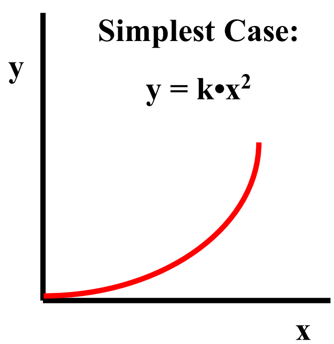
There are numerous patterns in Physics that could be described as displaying a quadratic relationship. When an equation is written for two quantities (call them x and y) that have a quadratic relationship, the highest exponent in the equation will be two (not 1, not 3, but 2). The equation relating x and y has the form $\mathbf{y = Ax^2 + Bx + C}$ where **A is a non-zero constant** and B and C can be any number (including 0). A plot of two quantities that have a quadratic relationship will be curved (not straight) and have the shape of a parabola. Often times, the constants B and C in the equation above are 0. In such cases, the equation becomes

$$\mathbf{y = Ax^2}$$

and the value of y is proportional to the square of the value of x. As such, a doubling of x causes a quadrupling of y. And a tripling of x causes a nine-fold increase in the value of y. In general terms, when the variable x is changed by some factor, the value of y will change by the square of that factor. An inspection of the x-y data table will often reveal such a pattern. By strategically comparing two rows of the data table, one will notice that if the value of x in one row is twice the value of x in another row, then the corresponding y value will be four times greater in one row than in the other row.

Simplest Case:

$\mathbf{y = k \cdot x^2}$



x	y
0	0
1	2
2	8
3	18
4	32
5	50

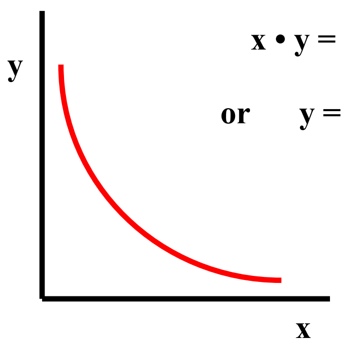
Quadratic (Simplest Case)

Double x … Quadruple y

Triple x … Nine times y

Inverse Relationships

Studies of variables in Physics will often show the pattern of an inverse relationship. When two variables (call them x and y for now) are inversely related, then an increase in one variable causes a decrease in the other variable ... and vice versa. An x-y graph of two inversely related quantities shows this pattern as a curved line that doesn't cross the axes. For such graphs, as x approaches large positive values, y approaches 0. The most general equation describing two inversely related quantities is the equation y = A/xn where A is a non-zero constant and n is some positive power. Often times in Physics, the power n is 1 and the equation becomes y = A/x where A is some non-zero constant. In this case, the equation can be rearranged to y•x = A and one can see that the product of x and y is always the same value. An inspection of the x-y data table will show that the produce of x and y is the same product for every row of the data table. For this to be the case, then a doubling of the value of x will result in the halving of the value of y. And a tripling of the value of x will result in thirding the value of y. And so on.



x • y = k

or y = k/x

x	y
1	12
2	6
3	4
4	3
5	2.4
6	2

Inverse

Double x … Halve y

Quadruple x … one-fourth y